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HEAVE AND PITCH MOTIONS OF A SHIP DUE TO MOVING MASSES AND FORCES

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The coupled heave and pitch motions of a ship floating on a still water surface and subjected to a moving mass have been investigated. The ship hull is considered as a rigid body with two degrees of freedom and supported by buoyancy force. Based on the general equations of ship dynamics, the governing equations of motion of the ship subjected to a moving mass are presented. The set of differential equations is then solved numerically. The calculated values of heave and pitch motions and contact force are compared with those obtained in the case of a moving force. Finally, the effect of the moving load velocity on the maximum heave and pitch motions of the ship is discussed.

1. INTRODUCTION

Analysis of the behaviour of structures subjected to various moving forces and masses is of great practical importance and therefore has attracted the attention of many researchers [1–5]. In most of the contributions to the subject, the vibration of an elastic structure, being subjected to some moving load, has been considered. However, in the case of a ship traversed by a moving load, which occurs during the take off of an aircraft from an aircraft carrier, rigid-body motions exist which have larger amplitudes than those which occur for elastic body motions [6].

While the estimation of the rigid-body motions of a ship moving in waves has been discussed in references [7–9], knowledge related to the moving force- (or mass-) induced motions of a ship on still water is essentially limited to a recent article by Wu and Sheu [10]. In the aforementioned paper, it has been assumed that the depth of water and the distance between the floating ship hull and the shore are so large that the most important effect due to ship-motion-induced waves is the damping effect and it may be replaced by the damping ratios. In such a case, the ship may be simulated as a rigid beam resting on an elastic foundation. The governing equations of motion of the ship hull subjected to a moving force were presented in reference [10] and an "exact solution" to these equations has been proposed. The proposed analytical solution is available only for the case of Rayleigh damping, and the present authors have previously discussed this matter in some detail [11].

In this paper, most of the assumptions made in reference [10] have been maintained and on this basis, the equations of motion for the dynamic behaviour of a ship due to a moving mass are derived. The major difference between the present analysis and that of reference [10] is that the ship is subjected to a moving mass (instead of a constant moving force). The contact force at the ship–load interface in the case of a moving mass is time-dependent and

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its variations over time have been evaluated in the following sections. The results obtained for the moving mass case have been compared with those corresponding to the moving force analysis. It may be concluded that the contact force due to a moving mass cannot be replaced with a constant moving force unless the moving mass is much smaller than the ship mass, and does not have high velocity.

In general, added mass and damping are frequency-dependent. As a result, the corresponding coefficients in the equations of motion would also depend on the frequency of motion. The consideration of such dependency would make the solution too sophisticated. Hence, in the following sections these coefficients will be treated as constant parameters having some average values. The equations of motion are then solved numerically for a typical problem and corresponding time histories of ship response and contact force are presented. Furthermore, the effect of the mass velocity on the maximum heave and pitch motions of the ship is studied. All of the results are compared with those obtained when the moving mass is substituted by a constant moving force, equal to the weight of the moving mass.

2. PROBLEM FORMULATION

In practice, a ship treated as a rigid body can experience all six-degrees-of-freedom (6-d.o.f.s) motions when it floats on the water surface. The corresponding equations of motion are coupled and highly non-linear. Studying this set of equations is difficult and alternative investigations are often restricted to the 2- or 3-d.o.f.s coupled motions. Among the latter, the heave and pitch motions can be investigated in a model basin.

The coupled heave and pitch motions in the case of head sea have been investigated analytically by applying the "strip theory". The method is based on Newton's second law and results in two differential equations for heave and pitch motions. The theoretical results obtained through the application of the strip theory have been verified experimentally [7].

Figure 1 illustrates the general arrangement of the ship and the moving mass. In this figure, some basic geometrical parameters of the ship and those related to the instantaneous position of the moving mass have been demonstrated. It should be noted that the ship model considered for performing calculations is not as complicated as that of Figure 1.

Figure 2 illustrates the xyz co-ordinate system which is stationary with its origin at the initial position of G (mass centre of the ship) before the load is imposed. The second co-ordinate system is attached to the ship and is denoted by $x'y'z'$. These two systems have also been considered by Nayfeh [12]. A third co-ordinate system $\bar{x}\bar{y}\bar{z}$, which can serve to express the position of the load with respect to stern, has also been used by some authors such as Wu and Sheu [10].

In order to reduce the amount of necessary mathematical manipulations, and to acquire relatively accurate results, the following set of assumptions is considered. First, the ship hull is assumed to have no forward speed and to be floating on a still water surface. Second, the ship-motion-induced waves are neglected. Third, the moving mass m is assumed to have the constant speed v_0 relative to the ship. Finally, the maximum pitch motion is assumed to be so small that there is no significant difference between normal and vertical directions and thus, the following approximations can be used: $\cos\theta \sim 1$ and $\sin\theta \sim \theta$.

The set of coupled differential equations of motion of the ship subjected to a moving mass and performing heave and pitch motions is [7, 10]

$$a\ddot{z} + b\dot{z} + cz - \mathrm{d}\ddot{\theta} - e\dot{\theta} - g\theta = P(t), \tag{1}$$

$$A\ddot{\theta} + B\dot{\theta} + C\theta - D\ddot{z} - E\dot{z} - Gz = P(t)\,(L_g - u), \tag{2}$$

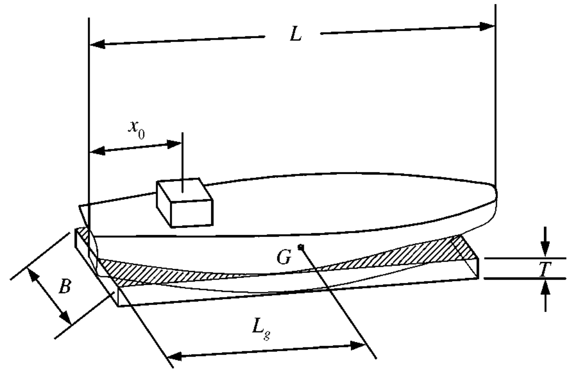

Figure 1. The general arrangement of the ship and the moving mass floating on still water.

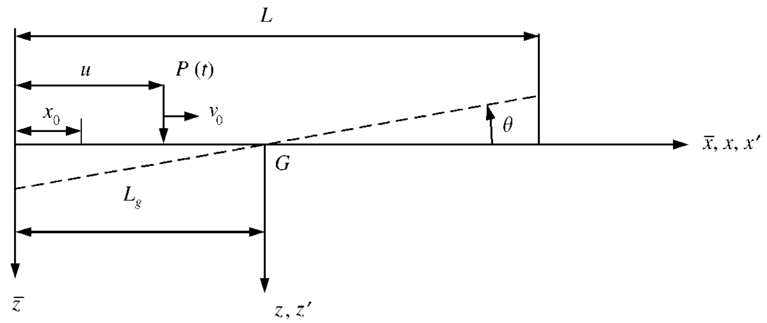

Figure 2. The co-ordinate systems for the heave and pitch motions.

where, $P(t)$ is the time-dependent contact force exerted by the moving mass on the ship, being equal to the value of the moving force in the case of a constant moving force. Therefore, the right-hand side of equations (1) and (2) demonstrate the contribution of the contact force in the force and moment equations.

Considering Figures 1 and 2, the following parameters may be defined: z is the vertical displacement of the mass centre of the ship hull (heave motion), θ is the angular displacement of the ship hull about y-axis (pitch motion), L_g is the distance between the mass centre of the ship hull and the stern, x_0 is the initial distance between the moving load and the stern, v_0 is the constant velocity of the moving mass, u is the instantaneous moving mass distance from the stern being equal to $x_0 + v_0 t$, L is the total length of the ship, and x is the longitudinal co-ordinate with origin amidships for the unloaded ship.

The coefficients on the left-hand side of equations (1) and (2) may be expressed as [7]

$$a = \int_L m' \,\mathrm{d}x' + \rho_w \nabla, \qquad b = \int_L N' \,\mathrm{d}x',$$

$$c = \int_L \rho_w g_0 B_1 \,\mathrm{d}x', \qquad d = \int_L m' x' \,\mathrm{d}x',$$

$$e = \int_L N' x' \,\mathrm{d}x', \qquad g = \int_L \rho_w g_0 B_1 x' \,\mathrm{d}x',$$

$$A = \int_L m' x'^2 \, dx' + k_{yy}^2 \rho_w \nabla, \qquad B = \int_L N' x'^2 \, dx', \tag{3}$$

$$C = \int_L \rho_w g_0 B_1 x'^2 \, dx', \qquad D = \int_L m' x' \, dx',$$

$$E = \int_L N' x' \, dx', \qquad G = \int_L \rho_w g_0 B_1 x' \, dx',$$

where, m' is the sectional added mass, N' is the sectional damping coefficient due to hydrodynamic resistance imposed on the ship, ∇ is the volume displacement of the ship hull, k_{yy} is the transverse radius of gyration of the ship hull, g_0 is the gravity acceleration of the Earth at the surface of free seas (subscript is used to avoid confusion with g as a ship parameter), ρ_w is the mass density of (fresh or sea) water, and B_1 is beam of the ship at each strip.

As mentioned earlier, $P(t)$ is not necessarily equal to the weight of the moving mass, but is the instantaneous contact force between the moving mass and the ship deck. To determine $P(t)$, the total vertical acceleration of the moving mass must be calculated.

Figure 3 illustrates the position vectors of the ship mass centre and that of the moving mass. It is assumed that the xyz co-ordinate system used is stationary with its origin at the initial position of G (mass centre of the ship), i.e., at $t = 0$, and $\mathbf{r}_m$ is the instantaneous position vector of the moving mass, $\mathbf{r}_M$ is the instantaneous position vector of mass centre of the ship (G). $\mathbf{h}$ is the instantaneous vector with origin at G and normal to the direction of motion of the moving mass and $\mathbf{i}$ and $\mathbf{k}$ are the unit vectors corresponding to x-and z-axis respectively.

The following relations may be drawn from Figure 3:

$$\mathbf{r}_m = \mathbf{r}_M + \mathbf{h} + (L_g - u)\,(-\cos\theta\mathbf{i} + \sin\theta\mathbf{k}), \tag{4}$$

$$\mathbf{r}_M = z\mathbf{k}, \qquad \mathbf{h} = -h\,(\sin\theta\mathbf{i} + \cos\theta\mathbf{k}). \tag{5, 6}$$

Equations (4)–(6) may be combined to yield the following equations for the components of $\mathbf{r}_m$:

$$x_m = -h\theta - (L_g - u), \qquad z_m = z - h + (L_g - u)\theta. \tag{7, 8}$$

Equations (7) and (8) are valid for small values of θ. Since the motion of the moving mass with respect to the ship has been assumed to be rectilinear and uniform ($u = x_0 + v_0 t$ and

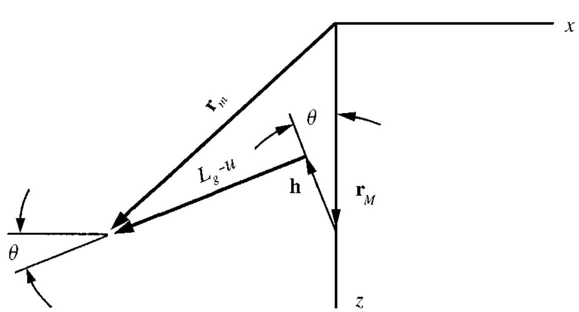

Figure 3. Relations between position vectors of the ship mass centre and that of the constant moving force.

h is considered to be constant) substitution in equations (7) and (8) and taking time derivatives results in

$$\dot{x}_m = -h\dot{\theta} + v_0, \qquad \dot{z}_m = \dot{z} + \dot{\theta}(L_g - u) - \theta v_0. \tag{9, 10}$$

In order to obtain absolute acceleration components of the mass m, equations (9) and (10) can be differentiated once again to get

$$\ddot{x}_m = -h\ddot{\theta}, \qquad \ddot{z}_m = \ddot{z} + \ddot{\theta}(L_g - u) - 2\dot{\theta}v_0. \tag{11, 12}$$

Considering Figure 4(a), the acceleration component normal to the ship deck, for small values of θ is

$$a_n = \ddot{z}_m + \ddot{x}_m\theta. \tag{13}$$

Substituting equations (11) and (12) into equation (13) yields

$$a_n = \ddot{z} - 2v_0\dot{\theta} + \ddot{\theta}(L_g - u) - h\theta\ddot{\theta}. \tag{14}$$

Considering the free-body diagram of the moving mass, i.e., Figure 4(b), the equation of motion of the moving mass in the normal direction would become for small θ,

$$mg_0 - P(t) = ma_n. \tag{15}$$

Substitution for a_n from equation (14) and solving for $P(t)$ gives

$$P(t) = m[g_0 - \ddot{z} + h\theta\ddot{\theta} + 2v_0\dot{\theta} - \ddot{\theta}(L_g - u)]. \tag{16}$$

Neglecting the term $h\theta\ddot{\theta}$ will result in the following expression for the contact force in terms of the motion of the ship and the moving mass:

$$P(t) = m[g_0 - \ddot{z} + 2v_0\dot{\theta} - \ddot{\theta}(L_g - u)]. \tag{17}$$

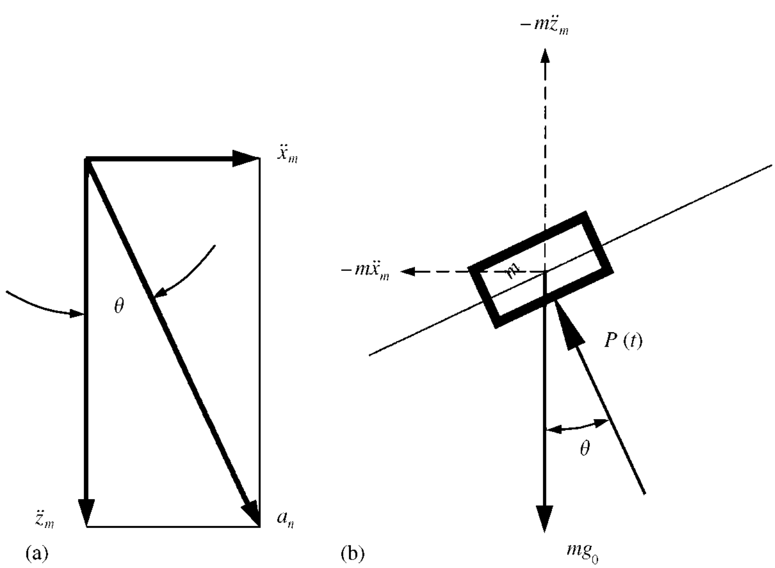

Figure 4.(a) Components of the moving mass acceleration. (b) Free-body diagram of the moving mass.

It can be shown that if the pitch angle is not assumed to be small at the early stages of the analysis, a somewhat different and of course, non-linear, expression for $P(t)$ would be obtained. However, it would once again reduce to equation (17) if the pitch angle is assumed to be small. Therefore, the final result is the same no matter where this assumption is implemented.

Substitution of $P(t)$ from equation (17) into equations (1) and (2), together with considering the identity $L_t = L_g - x_0 - v_0 t$ yields the set of equations of motion (as long as the load keeps contact with the ship) in the following final form:

$$(a+m)\ddot{z} + b\dot{z} + cz + (-d + mL_t)\ddot{\theta} + (-e - 2mv_0)\dot{\theta} - g\theta = mg_0, \quad (18)$$

$$(-D + mL_t)\ddot{z} - E\dot{z} - Gz + (A + mL_t^2)\ddot{\theta} + (B - 2mv_0 L_t)\dot{\theta} + C\theta = mg_0 L_t. \quad (19)$$

3. NUMERICAL SOLUTION

Since L_t is time-dependent, the system of equations (18) and (19) would have time-dependent coefficients and the problem at hand would be classified as a case of parametrically excited vibrations. The authors' attempts to solve equations (18) and (19) analytically, has not yet been successful. In what follows, these equations are solved numerically.

In order to present the results more efficiently, the basic variables have been normalized using the following characteristic values:

For $z(t)$; z_{st}, which is the initial value of the heave motion (i.e., at $t = 0$).

For $\theta(t)$; θ_{st} being the initial value of the pitch motion (i.e., at $t = 0$).

For t; $t_1 = (L - x_0)/v_0$, which is the time for the moving mass leaving the ship hull.

For $P(t)$; Mg_0 being the weight of the ship

For v_0; $\sqrt{Lg_0}$.

The static values of pitch and heave, i.e., z_{st} and θ_{st}, can be computed from equations (18) and (19) by the substitution of $P(t) = mg_0$, eliminating all time derivatives and solving for z and θ. The following set of linear equations can then be obtained:

$$cz_{st} - g\theta_{st} = mg_0, \qquad -Gz_{st} + A\theta_{st} = mg_0(L_g - x_0). \quad (20)$$

Equation (20) results in the following static heave and pitch values:

$$z_{st} = \frac{mg_0(gL_g - gx_0 + A)}{cA - Gg}, \qquad \theta_{st} = \frac{mg(G + cL_g - cx_0)}{cA - Gg}. \quad (21)$$

4. CASE STUDY

In order to perform the calculations and present the corresponding results, the model parameters used in Example 9·1 of reference [7] together with the set of parameters listed in Table 1 have been used. On pp. 191–198 of reference [7], the dynamic characteristics of the assumed model have been evaluated as

$$\begin{aligned}
&a = 164{\cdot}4\ \text{lbf s}^2/\text{ft}, \quad b = 106{\cdot}7\ \text{lbf s/ft}, \quad c = 2588{\cdot}5\ \text{lbf/ft},\\
&d = -14{\cdot}27\ \text{lbf s}^2, \quad e = -112{\cdot}73\ \text{lbf s}, \quad g = -1242{\cdot}5\ \text{lbf/ft},\\
&A = 3148{\cdot}7\ \text{lbf s}^2\text{ft}, \quad B = 2105{\cdot}9\ \text{lbf s ft}, \quad C = 48306{\cdot}24\ \text{lbf/ft},\\
&D = -14{\cdot}27\ \text{lbf s}^2, \quad E = -112{\cdot}73\ \text{lbf s}, \quad G = -1242{\cdot}5\ \text{lbf/ft}.
\end{aligned} \quad (22)$$

TABLE 1

Specifications of the ship model [7]

Length of model	19·2 ft
Maximum beam	2·592 ft
Draft	1·114 ft
Displacement	2837·76 lb
Block coefficient	0·8
LCG	0·48 ft
LCB	0·48 ft

Other parameters and initial conditions are as follows:

$$L = 19{\cdot}2\ \text{ft}, \quad L_g = 9{\cdot}6\ \text{ft}, \quad g_0 = 32{\cdot}2\ \text{ft/s}^2,$$
$$m = 10\ \text{slug}, \quad x_0 = 0, \quad v_0 = 20\ \text{ft/s}, \tag{23}$$
$$\dot{z}_0 = 0, \quad \dot{\theta}_0 = 0.$$

Parameters presented in equations (22) and (23) have been used to perform calculations. Figures 5 and 6 illustrate the normalized time histories of the ship heave and pitch motions due to the moving mass m and those obtained for a moving force equal to mg_0. On the horizontal axis, the instant corresponding to $t/t_1 = 1$ illustrates the time when the moving mass ceases to be in contact with the ship. Before t_1, forced vibrations occur and after t_1 free vibrations do take place.

By the comparison of the ship hull responses to a moving mass with responses to a moving force, it has been concluded that the maximum heave and pitch motions due to a moving force are larger. This result can be explained by noting that in the moving mass case, the normal contact force is not constant and is usually less than its initial value, thus causing vibrations with lower amplitudes. This description is verified by examining the time-history diagram of the contact force presented in Figure 7. As a matter of fact, it can be seen that the lower the magnitude of the contact force becomes, the greater deviation of the heave and pitch time histories due to the moving mass from those of the moving force is observed.

In order to analyze the speed effects, a computer program was developed and run several times with different values of the mass speed v_0 (with small increments). In this way, corresponding maximum values of heave and pitch motions were recorded. The results are presented in Figure 8 by solid line. From this figure it can be deduced that at certain values of the moving mass speed, maximum dynamic heave and pitch can exceed the static values. However, as the moving mass velocity exceeds a certain level, these maximum values of the dynamic response of the ship start to decrease. This finding is in good agreement with predictions based on physical sense. In this way, a specific speed can be introduced beyond which the maximum mass-induced displacements of the ship in heave and pitch motions are equal to or less than the corresponding static values.

In Figure 8, the dotted line corresponds to the moving force case. Because of the reduction of the amount of the contact force in the moving mass case, it is observed that the maximum heave motion becomes less than the corresponding heave motion when a constant moving force is considered.

The variation of maximum heave response of the ship with the speed of the moving mass is represented in Figure 9. However, in this figure, the mass of the load has been considered

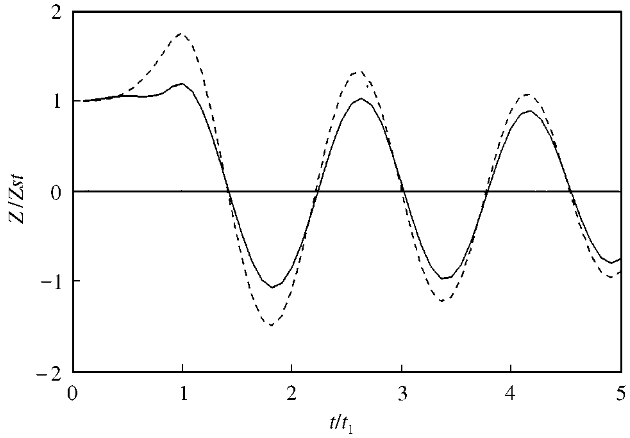

Figure 5. Time-history diagram of heave motion of the ship: (——), moving mass; and (– – – –), moving force.

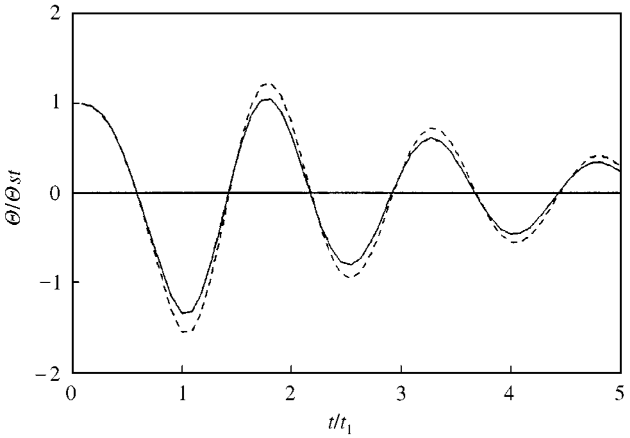

Figure 6. Time-history diagram of pitch motion of the ship: (——), moving mass; and (– – – –), moving force.

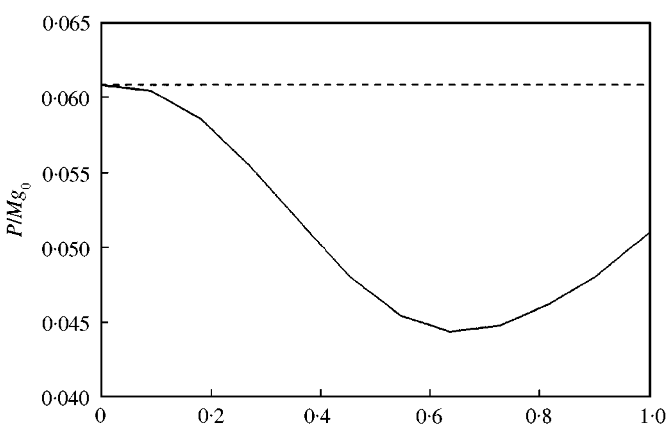

Figure 7 Time-history diagram of the contact force ($m/M = 0{\cdot}06$): (——), moving mass; and (– – – –), moving force.

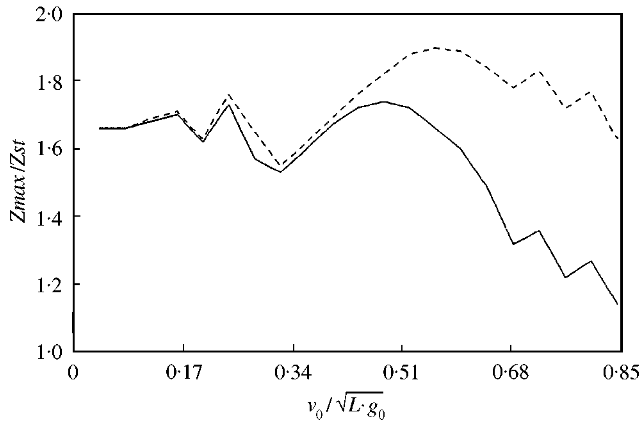

Figure 8. Variation of the maximum heave motion with the moving load velocity ($m/M = 0{\cdot}06$): (——), moving mass; and (– – – –), moving force.

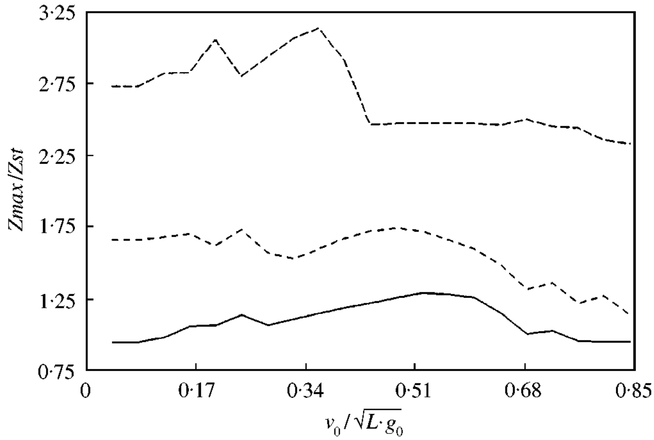

Figure 9. Variation of the maximum heave motion with the moving load velocity: (——), $m/M = 0{\cdot}03$; (- - - -), $m/M = 0{\cdot}06$; and (– – – –), $m/M = 0{\cdot}1$.

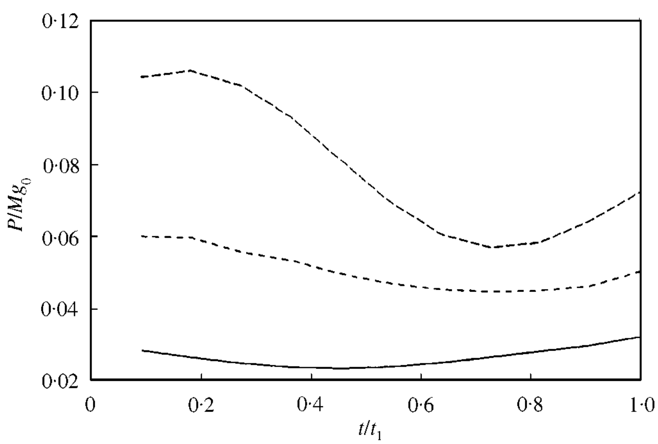

Figure 10. Time-history diagram of the contact force: (——), $m/M = 0{\cdot}03$; (- - - -), $m/M = 0{\cdot}06$; and (– – – –), $m/M = 0{\cdot}1$.

as another variable. The general pattern of the response curves is observed to be the same for various values of mass ratio. Furthermore, it is deduced that the increase of the mass of the load results in larger heave motions. It is also observed that when the mass ratio increases, the critical velocity (corresponding to maximum heave motion) decreases.

Figure 10 illustrates the variation of the contact force with time, until separation, for several mass ratios. It has been observed that in all cases the contact force has been less than the corresponding weight of the load. Furthermore, it can be observed that as the mass ratio increases, the percentage of maximum reduction of contact force also increases.

5. CONCLUSIONS

The coupled heave and pitch motions of a ship hull floating on a still water surface and subjected to a moving mass or a constant force were investigated. The general equations of the ship motions due to a moving load were obtained. The load–ship contact force was observed to be time-dependent and that it interacts dynamically with the ship motions until load separation occurs. Using normalized parameters, time histories of heave, pitch and contact force were compared with those of a constant moving force. It was observed that maximum dynamic responses corresponding to a moving mass are smaller than those for a constant moving force, being a result of contact force reduction. The effect of the load velocity on the maximum magnitude of heave motion of the ship was discussed. It was observed that after a threshold moving load velocity, the maximum heave and pitch responses of the ship decline and would no longer be larger than the corresponding static values.

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